Lernblatt Integrale

$$\int_a^b f(x)\,dx$$

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Friedrich Buckel

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	A Grundintegrale	B Wichtige Integrale	C Einfache Substitution od. Kettenregel	D Erweiterte Substitution	E Quadratische Substitution	F Besondere Intergrale
1	$\int_1^2 \frac{1}{x^2}dx$	$\int_1^2 \frac{1}{x}dx$	$\int_0^6 \frac{5}{(2x+4)^2}dx$	$\int_0^5 \frac{x}{x+4}dx$	$\int_0^2 \frac{x}{x^2+4}dx$	$\int_2^4 \frac{x^2-4}{2x}dx$
2	$\int_0^4 \sqrt{x}\,dx$	$\int_1^4 \frac{1}{\sqrt{x}}dx$	$\int_0^6 \sqrt{2x+4}\,dx$	$\int_0^5 x\sqrt{x+4}\,dx$	$\int_0^{\sqrt{5}} x\sqrt{x^2+4}\,dx$	$\int_0^4 (x+1)\sqrt{x}\,dx$
3	$\int_1^e \ln x\,dx$	$\int_1^e \frac{\ln x}{x}dx$	$\int_0^2 \ln(2x+4)\,dx$	$\int_0^5 x\cdot\ln(x+4)dx$	$\int_0^2 x\cdot\ln(x^2+4)dx$	$\int_1^e (\ln x)^2\,dx$
4	$\int_0^2 e^x dx$	$\int_0^2 \frac{1}{e^x}dx$	$\int_0^1 e^{2x+4}dx$	$\int_0^5 x\cdot e^{x+4}dx$	$\int_0^2 x\cdot e^{x^2+4}dx$	$\int_0^1 x^2\cdot e^x\,dx$
5	$\int_0^{\pi} \sin x\,dx$	$\int_0^{\frac{1}{2}\pi} \cos x\,dx$	$\int_0^{\pi} \sin\frac{x}{2}\,dx$	$\int_a^b x\cdot\sin x\,dx$	$\int_0^{\frac{1}{2}\pi} x\cdot(\sin x^2)dx$	$\int_0^{\pi} \sin^2 x\,dx$

Aufgabe: Trage in die nachfolgende leere Tabelle die ersten Schritte ein, eventuell Substitution oder partielle Integration.

	A Grundintegral	B Wichtiges Integral	C Substitution/Kettenreg.	D Erweiterte Substitution	E Quadrat. Substitution	F Besondere Intergrale
1	$= \int_1^2 x^{-2}dx = \left[\frac{x^{-1}}{-1}\right]_1^2$ $= \left[-\frac{1}{x}\right]_1^2 = -\frac{1}{2} + 1 = \frac{1}{2}$	$= [\ln\lvert x\rvert]_1^2 =$ $= \ln 2 - \underbrace{\ln 1}_{=0} = \ln 2$	Substitution: $u = 2x + 4$ $du = 2dx \Rightarrow dx = \frac{1}{2}du$ $= \frac{5}{2}\int_4^{16} \frac{1}{u^2}du = \frac{5}{2}\left[-\frac{1}{u}\right]_4^{16}$ oder: $= 5\left[\frac{(2x+4)^{-1}}{-1\cdot\boxed{2}}\right]_0^6$	Substitution: $u = x + 4$ $x = u - 4 \Rightarrow dx = du$ $= \int_4^9 \frac{u-4}{u}du = \int_4^9 \left(1 - \frac{4}{u}\right)du$ $= [u - 4\cdot\ln\lvert u\rvert]_4^9 = \dots$	Substitution: $u = x^2 + 4$ $du = 2x\cdot dx \Rightarrow x\cdot dx = \frac{1}{2}du$ $= \frac{1}{2}\int_4^8 \frac{1}{u}du = \frac{1}{2}[\ln\lvert u\rvert]_4^8$ $= \frac{1}{2}\ln 2$	Zerlegung in Einzelbrüche: $= \int_2^4 \left(\frac{1}{2}x - 2\cdot\frac{1}{x}\right)dx$ $= \left[\frac{1}{4}x^2 - 2\cdot\ln\lvert x\rvert\right]_2^4$
2	$= \int_0^4 x^{\frac{1}{2}}dx = \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^4$ $= \left[\frac{2}{3}x\sqrt{x}\right]_0^4 = \frac{16}{3}$	$= \int_1^4 x^{-\frac{1}{2}}dx = \left[\frac{x^{\frac{1}{2}}}{\frac{1}{2}}\right]_1^4$ $= [2\sqrt{x}]_1^4 = 4 - 2 = 2$	Substitution: wie C1 $= \frac{1}{2}\int_4^{16} u^{\frac{1}{2}}du = \frac{1}{2}\left[\frac{2}{3}u\sqrt{u}\right]_4^{16}$ oder: $\left[\frac{(2x+4)^{3/2}}{\frac{3}{2}\cdot\boxed{2}}\right]_0^6$	Substitution: $u = \sqrt{x+4}$ $x = u^2 - 4 \Rightarrow dx = 2u\cdot du$ $= 2\int_2^3 (u^2 - 4)u\,du$ $= 2\int_4^9 (u^3 - 4u)\,du = \dots$	Substitution: siehe E1 $= \frac{1}{2}\int_4^9 \sqrt{u}\,du = \frac{1}{2}\cdot\frac{2}{3}\cdot[u\sqrt{u}]_4^9$ Siehe A2	In Potenzform bringen $= \int_0^4 \left(x^{\frac{3}{2}} + x^{\frac{1}{2}}\right)dx$ $= \left[\frac{2}{5}x^2\sqrt{x} + \frac{2}{3}x\sqrt{x}\right]_0^4$
3	$= [x\cdot\ln x - x]_1^e$ $= e\cdot\underbrace{\ln e}_{1} - e - 1\cdot\underbrace{\ln 1}_{0} + 1 = 1$	Substitution: $u = \ln x \Rightarrow du = \frac{1}{x}dx$ $= \int_0^1 u\,du = \left[\frac{1}{2}u^2\right]_0^1 = \frac{1}{2}$	Substitution: wie C1 $= \frac{1}{2}\int_4^8 \ln u\,du = \frac{1}{2}[u\cdot\ln u - u]_4^8$ od. $= \left[\frac{(2x+4)\cdot\ln(2x+4) - 2x - 4}{\boxed{2}}\right]_0^2$	Substitution: wie D1 $= \int_4^9 (u - 4)\cdot\ln u\,du$ Partielle Integration: $v' = u - 4 \Rightarrow v = \frac{1}{2}u^2 - 4u$ $w = \ln u \Rightarrow w' = \frac{1}{u}$	Substitution: siehe E1 $= \frac{1}{2}\int_4^8 \ln u\,du = \frac{1}{2}[u\cdot\ln u - u]_4^8$	Partielle Integration: $u' = \ln x \Rightarrow u = x\cdot\ln x - x$ $v = \ln x \Rightarrow v' = \frac{1}{x}$ oder: $u' = 1 \Rightarrow u = x$ $v = (\ln x)^2 \Rightarrow v' = 2\ln x\cdot\frac{1}{x}$
4	$= [e^x]_0^2 = e^2 - 1$	$= \int_0^2 e^{-x}dx = [-e^{-x}]_0^2$ $= 1 - e^{-2}$	Ohne Substitution: $= \left[\frac{1}{2}e^{2x+4}\right]_0^1 = \frac{1}{2}(e^6 - e^4)$ oder mit Substitution wie C1	Partielle Integration: $v' = e^{x+4} \Rightarrow v = e^{x+4}$ $w = u \Rightarrow w' = 1$ $= [xe^{x+4}]_0^5 - \int_0^5 e^{x+4}dx$	Substitution: siehe E1 $= \frac{1}{2}\int_4^8 e^u du = \frac{1}{2}[e^u]_4^8 = \dots$	2 mal Partielle Integration $u' = e^x \Rightarrow u = e^x$ $v = x^2 \Rightarrow v' = 2x$ dann $u' = e^x$, $v = x$.
5	$= [-\cos x]_0^{\pi} = 2$	$= [\sin x]_0^{\frac{1}{2}\pi} = 1$	Sub:: $u = \frac{x}{2} \Rightarrow x = 2u \Rightarrow dx = 2du$ $2\int_0^{\frac{1}{2}\pi} \sin u\,du = 2[-\cos u]_0^{\frac{1}{2}\pi}$ od. $= \left[\frac{\cos(x/2)}{1/2}\right]_0^{\pi} = \left[2\cdot\cos\left(\frac{x}{2}\right)\right]_0^{\pi}$	Partielle Integration: $u' = \sin x \Rightarrow u = -\cos x$ $v = x \Rightarrow v' = 1$ $= [-x\cos x]_a^b + \int_a^b \cos x\,dx$	Substitution: $u = x^2$ $du = 2x\cdot dx \Rightarrow x\cdot dx = \frac{1}{2}du$ $= \frac{1}{2}\int_c^d \sin u\,du = \frac{1}{2}[-\cos u]_c^d$	Partiell mit Trick (lernen !) oder ersetzen: $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$ dann in 2 Integrale teilen.

	A	B	C	D	E	F
1						
2						
3						
4						
5						